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Order $\alpha_s(Q^2)$ QCD corrections to the polarised $e^+ e^- \rightarrow \Lambda X$

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Abstract

The importance of polarised gluons fragmenting into Λ in the polarised e^+e^- annihilation is discussed using the Altarelli-Parisi evolution equations satisfied by the quark and gluon fragmentation functions. In this context, the polarised fragmentation function $\hat{g}_1^\Lambda(x, Q^2)$ appearing in the cross section is discussed within the parton model. We relate this fragmentation function to the quark, anti-quark and gluon fragmentation functions and also find the QCD corrections to order α_s .

Quantum Chromodynamics(QCD) has been the most successful theory of strong interaction physics. A number of Deep Inelastic Scattering(DIS) experiments not only tested the excellent predictions of QCD such as scaling violation but also unravelled the structure of the hadrons in terms of its fundamental constituents such as quarks and gluons. The careful study of the unpolarised lepton proton DIS showed that the gluons carry significant amount of momentum of the proton. This is an interesting result as it contradicts our naive expectation that the gluonic contribution is next to leading order effect. This naive expectation is due to simple observation that one has to pay price for the strong coupling constant when photon probes the gluonic content of the proton. This simple argument based on strong coupling constant fails due to the appearance of large logarithms at very high energies. More sophisticated analysis based on the Altarelli-Parisi(AP) evolution equations satisfied by the parton probability distribution functions confirms the importance of the role played by the gluons [1].

More recently, a series of DIS experiments with polarised beam and polarised target helped us to understand the spin structure of the proton. These experiments measured the polarised structure function $g_1(x, Q^2)$ of the proton. Here x is the usual Bjorken variable and Q^2 is the invariant mass of the probing photon. The first moment of this structure function directly measures the spin contribution coming from the constituents of the polarised proton. To lowest order,

$$\int_0^1 g_1(x, Q^2) dx = \frac{1}{2} \sum_q e_q^2 [\Delta q(Q^2) + \Delta \bar{q}(Q^2)] \quad (1)$$

where e_q is the charge of the quark q and $\Delta q(Q^2) = \int_0^1 \Delta q(x, Q^2) dx$. Using the low energy data on $\Delta u(Q^2) - \Delta d(Q^2)$ and $\Delta u(Q^2) + \Delta d(Q^2) - 2\Delta s(Q^2)$, the measured first moment predicts that the spin contribution coming from the quarks is small. The naive argument based on the fact that the gluons are next to leading order again fails and in fact the gluonic contribution to the first moment of $g_1(x, Q^2)$ is found to be large. The reason for this is that the AP evolution equation for the polarised gluon distribution function shows that $\alpha_s(Q^2)\Delta g(Q^2)$ is scale independent to order $\alpha_s(Q^2)$ (strong coupling constant, $\alpha_s(Q^2) = g_s^2/4\pi$) [2]. Crudely speaking, large logarithms appearing in the high energy scattering cross sections and the running of strong coupling constant invalidate our naive expectations based on simple counting rules.

Hadron production in e^+e^- annihilation plays the crucial role as DIS does to understand the structure of the hadron. DIS experiments help us to understand how quarks and gluons share the properties of hadron such as its momentum and its spin. On the other hand, hadroproduction in e^+e^- annihilation is useful to understand the fragmentation mechanism of quarks and gluons into hadrons [3]. There has been a number of experiments on unpolarised e^+e^- experiments and a lot of theoretical works to understand the data [4]. All these analysis show that QCD corrections are very important. More recently, a series of experiments have been done at CERN measuring polarisation of hyperons produced [5]. On the theoretical side, there has been a lot of interesting developments. Burkardt and Jaffe [6] have proposed an experimental programme to successfully measure the polarised fragmentation functions. This programme requires the measurement of the total inclusive Λ production in polarised e^+e^- annihilation at various energies.

Recently a systematic analysis [7] of the evolution of polarised quark and gluon fragmentation functions has been done using the AP evolution equations satisfied by these polarised fragmentation functions. The analysis showed that the polarised gluon fragmentation is as significant as quark fragmentation functions. So in this letter we calculate the QCD corrections to polarised e^+e^- scattering to order $\alpha_s(Q^2)$ in the parton model. This consists of two sectors. 1. Quark sector and 2. Gluon sector. In the quark sector, the quark and/or anti quark is polarised. They are QCD corrected by gluon bremsstrahlung in addition to virtual corrections due to gluons. This changes the probability of a polarised quark and/or antiquark fragmenting into a polarised hadron. In the gluon sector, the contribution starts at order $\alpha_s(Q^2)$. This in fact measures the probability of polarised gluon fragmenting into polarised hadron. All these processes to order $\alpha_s(Q^2)$ are singular in both soft and collinear limit when the masses of quarks are taken to be zero. We regulate them by giving a small mass m_g to gluons. Hence, the polarised fragmentation functions appearing in the cross section formula are defined in this scheme. We work in the "massive gluon scheme" as it regulates both soft and collinear divergences simultaneously. The other reason why we work in the massive gluon scheme is that in the polarised DIS experiments, the data can be better understood in terms of polarised gluon distributions defined in the massive gluon scheme. It is worth recalling that the total cross section measured in the laboratory is nothing to do with the scheme we choose. Hence, the choice is immaterial as far as the physical predictions are con-

cerned. In the quark sector we encounter Ultraviolet(UV) divergences too. We regulate them in the well known gauge invariant Pauli-Villar's regularisation. All these divergences are shown to cancel among themselves, thanks to Ward Identity. Since we set quark masses to be zero, the convenient basis is helicity basis.

Let us first consider the AP evolution equation satisfied by the polarised quark and gluon fragmentation functions.

$$\frac{d}{dt} \begin{pmatrix} \Delta D_{q_i}^H(x, t) \\ \Delta D_g^H(x, t) \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} \begin{pmatrix} \Delta P_{qq}(x/y) & \Delta P_{gq}(x/y) \\ \sum_q \Delta P_{qg}(x/y) & \Delta P_{gg}(x/y) \end{pmatrix} \begin{pmatrix} \Delta D_{q_i}^H(y, t) \\ \Delta D_g^H(y, t) \end{pmatrix} \quad (2)$$

where $\Delta P_{ij}(x)$ are given in [1] and $t = \log(Q^2/\Lambda^2)$. The matrix ΔP is just the transpose of what is appearing in the AP equation for polarised parton distribution functions. Noting that the first moments of $\Delta P_{qq}(z)$ and $\Delta P_{gq}(z)$ are zero and that of $\Delta P_{qg}(z)$ and $\Delta P_{gg}(z)$ are 2 and $2\pi\beta_0$ (where $\beta_0 = (11C_2(G) - 4T(R))/12\pi$ with $C_2(R) = 4/3$, $C_2(G) = 3$ and $T(R) = f/2$, f being the number of flavours) respectively, we find that the first moment of polarised quark and gluon fragmentation functions satisfy the following simple differential equations.

$$\frac{d}{dt} \Delta D_g^H(t) = \alpha_s(t) \beta_0 \Delta D_g^H(t) \quad (3)$$

$$\frac{d}{dt} \Delta D_q^H(t) = \frac{1}{\pi} \alpha_s(t) \Delta D_g^H(t) \quad (4)$$

The solution to these equations can be found very easily. It turns out the polarised gluon fragmentation function satisfies

$$\frac{d}{dt} (\alpha_s(t) \Delta D_g^H(t)) = 0 (\alpha_s(t)^3) \quad (5)$$

where renormalisation group equation for $\alpha_s(t)$ i.e $d\alpha_s(t)/dt = -\beta_0\alpha_s(t)^2$ has been used. This is analogous to the differential equation satisfied by the first moment of polarised gluon distribution function, i.e $d\alpha_s(t)\Delta g(t)/dt = 0$ to order $\alpha_s(t)$ where $\Delta g(t)$ is the first moment of polarised gluon distribution function [2]. From eqn.(5), we find that $\alpha_s(t)\Delta D_g^H(t)$ is constant. This implies that the first moment of polarised gluon fragmentation function grows logarithmically. Using this, one finds that $\Delta D_q^H(t)$ is proportional to t . This analysis implies that when one wants to understand how much of the spin

is transferred to hadron, one must consider the direct gluon contribution also. It is in this spirit we compute the gluonic contribution to polarised e^+e^- scattering. In particular, we consider the inclusive polarised Λ hyperon production in the annihilation of polarised e^+e^- process. The reason why we consider this is two fold. Firstly the inclusive Λ production cross section is large compared to other particles and secondly it is easy to measure the polarisation of Λ . Our analysis can be extended to other hadro productions. We work in the energy region where only photon channel is dominant. The complete analysis including Z exchange channel is reserved for future publication [8].

We consider the inclusive polarised Λ production rate which factorises as:

$$d\sigma(s, s_1) = \frac{1}{4q_1 \cdot q_2} L^{\mu\nu}(q_1, q_2, s_1) \left(\frac{e^2}{Q^4} \right) 4\pi W_{\mu\nu}^\Lambda(q, p, s) \frac{d^3p}{(2\pi)^3 2p_0} \quad (6)$$

where $L_{\mu\nu}(q_1, q_2, s_1)$ is the leptonic part arising from e^+e^- annihilation into a photon of virtuality Q^2 and $W_{\mu\nu}^\Lambda(p, q, s)$ is photon fragmentation tensor. The first two arguments of these tensors are momenta described in the figures 1,2,3 and s, s_1 are the spins of the Λ and the electron respectively. The photon fragmentation tensor, sometimes called hadronic tensor contains all the informations about the polarised quark and gluon fragmenting into hyperons.

The operator definition of $W_{\mu\nu}^\Lambda(q, p, s)$ is found to be

$$W_{\mu\nu}^\Lambda(q, p, s) = \frac{1}{4\pi} \int d^4\xi e^{iq \cdot \xi} \langle 0 | J_\mu(0) | \Lambda(p, s) X \rangle \langle \Lambda(p, s) X | J_\nu(\xi) | 0 \rangle \quad (7)$$

where $J_\mu(\xi)$ is the electromagnetic(em) current, X is unobserved hadrons (summation over X is implicit), p and s are the momentum and spin of the Λ detected. Since we are interested in the polarised cross section and the energy is not too high to produce Z vector boson, only antisymmetric part of leptonic and photonic tensors contribute to this cross section. The antisymmetric parts of the leptonic tensor is found to be

$$L_{\mu\nu}(q_1, q_2, s_1) = -2ie^2 s_1 \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \quad (8)$$

where q_1, q_2 are the momenta of the incoming leptons and s_1 is the spin of the polarised lepton(electron). On the other hand the photonic tensor is not calculable in Perturbative QCD(PQCD) as we do not know how to compute

the matrix element of *em* current between hadronic states and the vacuum. But this can be parametrised using Lorentz covariance, gauge invariance, Hermiticity, and parity invariance. The anti symmetric part of this tensor takes the following form [9]:

$$W_{\mu\nu}^\Lambda(q, p, s) = \frac{i}{p \cdot q} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma \hat{g}_1^\Lambda(x, Q^2) + \frac{i}{p \cdot q} \epsilon_{\mu\nu\lambda\sigma} q^\lambda \left(s^\sigma - \frac{s \cdot q}{p \cdot q} p^\sigma \right) \hat{g}_2^\Lambda(x, Q^2) \quad (9)$$

where $x = 2p \cdot q / Q^2$, $Q^2 = q^2$ and $s^2 = -1$. Here the polarised fragmentation functions $\hat{g}_i^\Lambda(x, Q^2)$ are real and Lorentz invariant, hence they are functions of x and Q^2 . We have put hat on them to distinguish them from the polarised structure functions appearing in the polarised DIS. The following asymmetric cross section projects out only the $\hat{g}_1^\Lambda(x, Q^2)$ structure function as

$$\frac{d\sigma(\uparrow\uparrow - \uparrow\downarrow)}{dx d\cos(\theta)} = \alpha^2 \frac{\pi}{Q^2} x \hat{g}_1^\Lambda(x, Q^2) \cos(\theta) \quad (10)$$

where $\alpha = e^2/4\pi$, θ is the angle between the produced Λ particle and the incoming electron. Here, $\uparrow\uparrow$ means that both incoming electron and the produced hardron are paralelly polarised and $\uparrow\downarrow$ means that they are polarised antiparalelly. Note that $\hat{g}_2^\Lambda(x, Q^2)$ does not contribute to this asymmetry. The above cross section is zero when we integrate over θ . This implies that the polarised fragmentation function $\hat{g}_1^\Lambda(x, Q^2)$ can be measured only through angular distribution of the polarised hyperon. In the following, we interpret this structure function in the parton model in terms of polarised quark and gluon fragmentation functions [3].

In the parton model the Λ production inclusive cross section can be expressed in terms of inclusive quark and gluon production cross sections convoluted with appropriate quark and gluon fragmentation functions.

$$\frac{d\sigma(s, s_1)}{dx d\cos(\theta)} = \sum_{a,h} \int_x^1 \frac{dy}{y} \frac{d\hat{\sigma}^{a(h)}(s_1)}{dy d\cos(\theta)} D_{a(h)}^{\Lambda(s)}(x/y, Q^2) \quad (11)$$

where a runs over all the partons such as quarks, antiquarks and gluons and h is their helicity. Here the left hand side is the parton differential cross section for the production of parton of type a with polarisation h and energy fraction $y = 2p \cdot q / Q^2$ produced at an angle θ with respect to the beam direction(electron). $D_{a(h)}^{\Lambda(s)}(z, Q^2)$ is the probability of a parton of type a with

polarisation h fragmenting into Λ with polarisation s and the momentum fraction z of the parent parton. In the parton model, the asymmetry we consider in eqn.(10) turns out to be

$$\frac{d\sigma(\uparrow\uparrow - \uparrow\downarrow)}{dx d\cos(\theta)} = \sum_a \int_x^1 \frac{dy}{y} \frac{d\Delta\hat{\sigma}^a}{dy d\cos(\theta)} \Delta D_a^\Lambda(x/y, Q^2) \quad (12)$$

where

$$\begin{aligned} \frac{d\Delta\hat{\sigma}^a}{dx d\cos(\theta)} &= \frac{d\hat{\sigma}^{a(\uparrow)}(\uparrow)}{dx d\cos(\theta)} - \frac{d\hat{\sigma}^{a(\downarrow)}(\uparrow)}{dx d\cos(\theta)} \\ \Delta D_a^\Lambda(z, Q^2) &= D_{a(\uparrow)}^\Lambda(z, Q^2) - D_{a(\downarrow)}^\Lambda(z, Q^2) \end{aligned}$$

To arrive at this simple form, we have used parity invariance of the fragmentation functions.

In general $\hat{g}_1^\Lambda(x, Q^2)$ can be decomposed into the following parts:

$$\hat{g}_1^\Lambda(x, Q^2) = \hat{g}_1^0(x, Q^2) + \hat{g}_1^q(x, Q^2) + \hat{g}_1^g(x, Q^2) \quad (13)$$

The first term is the lowest order contribution coming from Fig. 1. The second term $\hat{g}_1^q(x, Q^2)$ is $\alpha_s(Q^2)$ correction coming from gluon bremsstrahlung and virtual corrections to the polarised quark and anti quark fragmenting into a polarised Λ . The third term comes from the polarised gluon fragmenting into a polarised Λ . The lowest order term $\hat{g}_1^0(x, Q^2)$ can be computed from the Fig. 1. The asymmetry to lowest order is found to be

$$\frac{d\sigma^0(\uparrow\uparrow - \uparrow\downarrow)}{dz d\cos(\theta)} = 3\alpha^2 e_q^2 \frac{\pi}{Q^2} \delta(1-z) \cos(\theta) \quad (14)$$

where e_q is the charge of the quark. This is convoluted with appropriate quark and antiquark polarised fragmentation functions to get $\hat{g}_1^0(x, Q^2)$ (see parton model expression eqn.(12)):

$$\hat{g}_1^0(x, Q^2) = 3 \frac{1}{x} \sum_q e_q^2 \left(\Delta D_q^\Lambda(x, Q^2) + \Delta D_{\bar{q}}^\Lambda(x, Q^2) \right) \quad (15)$$

Next we compute $\hat{g}_1^q(x, Q^2)$. We first compute the real gluon emission i.e bremsstrahlung contribution to it. These processes are both soft and collinear singular when we keep all the masses zero. In order to regulate these two

divergences simultaneously we give a small mass m_g to the gluons. The gluon bremsstrahlung contribution to this process can be formally expressed as

$$\frac{d\sigma^q}{dx_1 d\cos(\theta)} = \frac{is}{4q_1 \cdot q_2} L_{\mu\nu}(q_1, q_2, s_1) \frac{1}{Q^4} \epsilon_{\mu\nu\lambda\sigma} q^\lambda p_1^\sigma \frac{Q}{p_1 \cdot q} H_q(p_1, q) \quad (16)$$

where $x_1 = 2p_1 \cdot q / Q^2$, p_1 is the momentum of the polarised anti-quark, $q = q_1 + q_2$, θ is the angle between produced anti-quark and the incoming electron and s is its spin. The projected hard part of the Fig. 2, $H_q(p_1, q)$ is given by

$$H_q(p_1, q) = \frac{Q}{32(2\pi)^3} \int dx_2 \mathcal{P}_q^{\mu\nu} |M^q|_{\mu\nu}^2 \quad (17)$$

where $x_2 = 2p_2 \cdot q / Q^2$ and the projector $\mathcal{P}_q = i\epsilon_{\mu\nu\lambda\sigma} p_1^\lambda q^\sigma / 2p_1 \cdot q$. In terms of Mandelstam variables, the projected matrix element square $\mathcal{P}_q \cdot |M^q|^2$ is found to be

$$\begin{aligned} \mathcal{P}_q^{\mu\nu} |M^q|_{\mu\nu}^2 &= \frac{32(2\pi)^3 e_q^2 \alpha \alpha_s}{\pi} \left[2 \frac{s+t-m_g^2-Q^2}{st(Q^2-t)} (st + Q^2 t - Q^4 - m_g^2 Q^2) \right. \\ &\quad \left. + \frac{st - m_g^2 Q^2}{t^2} + \frac{st - m_g^2 Q^2}{s^2(t-Q^2)} (t + 2s - 2m_g^2 - Q^2) \right] \end{aligned} \quad (18)$$

where the Mandelstam variables $s = (p_1 + p_3)^2$, $t = (p_2 + p_3)^2$ and $u = (p_1 + p_2)^2$ satisfy $s + t + u = m_g^2 + Q^2$. Noting that $s = Q^2(1 - x_2)$ and $t = Q^2(1 - x_1)$ and using eqns.(16,17,18), the differential cross section for the emission of real gluons is found to be

$$\begin{aligned} \frac{d\sigma^q}{dx_1 d\cos(\theta)} &= \frac{2e_q^2 \alpha^2 \alpha_s}{Q^2} \left[\left(\frac{1+x_1^2}{1-x_1} \right) \log \left(\frac{Q^2 x_1 (1-x_1)}{m_g^2} \right) - \frac{3}{2} (1-x_1) \right. \\ &\quad \left. - \frac{3}{2} \frac{1}{(1-x_1)} + \frac{5}{4} \delta(1-x_1) \right] \cos(\theta) \end{aligned} \quad (19)$$

Note that there are two types of singularities appearing in the expression when we take $x_1 \rightarrow 0$ and $m_g \rightarrow 0$.

Next we compute the virtual gluon contribution to this differential cross section. It involves the evaluation of self energy and vertex corrections (see Fig. 3). Both diagrams in Fig. 3 are UV divergent separately. As we have already mentioned, we regulate these UV divergences in Pauli-Villars's

regularisation scheme. They also suffer from soft divergences when we set all the masses to be zero. Here also, we give a small mass to the gluon to regulate them. The amplitude for the self energy insertion after properly taking care of wave function renormalisation turns out to be (self energies for both quark and antiquark)

$$M_\mu^{self} = \frac{i}{\pi} e_q e \alpha_s \log \left(\frac{L}{m_g^2} \right) \bar{v}_s(p_1) \gamma_\mu u_{s'}(p_2) \quad (20)$$

where L is the Pauli-Villars's regulator to regulate the UV divergence. s' and s are the polarisations of quark and antiquark respectively.

Similarly the amplitude for the vertex correction turns out to be

$$\begin{aligned} M_\mu^{vertex} = & \frac{i}{\pi} e_q e \alpha_s \left[\log^2 \left(\frac{m_g^2}{Q^2} \right) + 3 \log \left(\frac{m_g^2}{Q^2} \right) \right. \\ & \left. + \frac{7}{2} - \frac{\pi^2}{3} - \log \left(\frac{L}{m_g^2} \right) \right] \bar{v}_s(p_1) \gamma_\mu u_{s'}(p_2) \end{aligned} \quad (21)$$

Though the above amplitudes suffer from UV divergence, when you sum these two amplitudes, we find that the UV regulator cancels between self energy and vertex corrections, thanks to Ward identity. Hence the virtual contribution to the partial differential cross section is found to be

$$\begin{aligned} \frac{d\sigma^v(\uparrow\uparrow - \uparrow\downarrow)}{dx_1 d\cos(\theta)} = & \frac{2e_q^2 \alpha^2 \alpha_s}{Q^2} \left[-\log^2 \left(\frac{m_g^2}{Q^2} \right) - 3 \log \left(\frac{m_g^2}{Q^2} \right) \right. \\ & \left. - \frac{7}{2} + \frac{\pi^2}{3} \right] \delta(1 - x_1) \cos(\theta) \end{aligned} \quad (22)$$

Adding the virtual and bremsstrahlung gluon contributions we obtain

$$\begin{aligned} \frac{d\sigma^q(\uparrow\uparrow - \uparrow\downarrow)}{dx_1 d\cos(\theta)} = & \frac{2e_q^2 \alpha^2 \alpha_s}{Q^2} \left[\left(\frac{1+x_1^2}{1-x_1} \right)_+ \log \left(\frac{Q^2}{m_g^2} \right) - \frac{3}{2}(1-x_1) \right. \\ & + (1+x_1^2) \left(\frac{\log(1-x_1)}{1-x_1} \right)_+ + \frac{1+x_1^2}{1-x_1} \log(x_1) \\ & \left. - \frac{3}{2} \left(\frac{1}{1-x_1} \right)_+ - \left(\frac{9}{4} - \frac{\pi^2}{3} \right) \delta(1-x_1) \right] \cos(\theta) \end{aligned} \quad (23)$$

where the $+$ prescription is defined in the usual way. That is,

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx \frac{f(x) - f(1)}{1-x} \quad (24)$$

for any smooth function $f(x)$. Notice that the eqn.(23) is free of any soft singularity in the limit $m_g \rightarrow 0$. The only divergent present in the above expression in this limit is collinear divergent. This divergent term is absorbed into the polarised anti-quark fragmentation function and hence the fragmentation function is defined in the massive gluon scheme. Similarly, one can compute the gluonic contribution when the quark is polarised. We find this contribution is same as that of anti-quark. Comparing the eqn.(23) with eqn.(10) and including gluonic contribution to polarised quark, we find

$$\hat{g}_1^q(x, Q^2) = 3 \frac{1}{x} \sum_q e_q^2 \int_x^1 \frac{dy}{y} \mathcal{C}_q(y, Q^2) [\Delta D_q^\Lambda(x/y, Q^2) + \Delta D_{\bar{q}}^\Lambda(x/y, Q^2)] \quad (25)$$

where

$$\begin{aligned} \mathcal{C}_q(y, Q^2) = & \frac{4}{3} \frac{\alpha_s}{2\pi} \left[\left(\frac{1+y^2}{1-y} \right)_+ \log \left(\frac{Q^2}{\Lambda^2} \right) + (1+y^2) \left(\frac{\log(1-y)}{1-y} \right)_+ \right. \\ & + \left(\frac{1+y^2}{1-y} \right) \log(y) - \frac{3}{2}(1-y) - \frac{3}{2} \left(\frac{1}{1-y} \right)_+ \\ & \left. - \left(\frac{9}{4} - \frac{\pi^2}{3} \right) \delta(1-y) \right] \quad (26) \end{aligned}$$

Now we turn to the computation of the cross section when the gluon is polarised. This amplitude is different from the amplitude for polarised quark. The differential cross section for the production of polarised gluon can be formally written as

$$\frac{d\sigma^g}{dx_3 d\cos(\theta)} = \frac{is}{4q_1 \cdot q_2} L_{\mu\nu}(q_1, q_2, s_1) \frac{1}{Q^4} \epsilon_{\mu\nu\lambda\sigma} q^\lambda p_3^\sigma \frac{Q}{p_3 \cdot q} H_g(p_3, q) \quad (27)$$

where $x_3 = p_3 \cdot q / Q^2$, p_3 is the momentum of the polarised gluon and θ is the angle between produced polarised gluon and the incoming electron. The hard part of the polarised gluon emission $H_g(p_3, q)$ is given by

$$H_g(p_3, q) = \frac{Q}{32(2\pi)^3} \int dx_2 \mathcal{P}_g^{\mu\nu} |M^g|_{\mu\nu}^2 \quad (28)$$

where the projector $\mathcal{P}_g^{\mu\nu} = i\epsilon_{\mu\nu\lambda\sigma}p_3^\lambda q^\sigma/2p_3 \cdot q$. The projected matrix element square $\mathcal{P}_g \cdot |M^g|^2$ is computed from the Fig. 2 (with gluon polarised) and is given by

$$\begin{aligned} \mathcal{P}_g^{\mu\nu} |M^g|_{\mu\nu}^2 = & \frac{16(2\pi)^3 e_q^2 \alpha \alpha_s}{\pi} \left(\frac{m_g^2 Q^2 - st}{(s+t)^2} \right) \left[2 \frac{(s+t)(s+t-m_g^2-Q^2)}{st} \right. \\ & + \frac{4m_g^2 Q^2 - 2m_g^2 s - 2Q^2 s + s^2 - t^2}{t^2} \\ & \left. + \frac{4m_g^2 Q^2 - 2m_g^2 t - 2Q^2 t + t^2 - s^2}{s^2} \right] \end{aligned} \quad (29)$$

Following the similar procedure adopted in the quark sector, we obtain the expression for differential cross section for polarised gluon emission:

$$\frac{d\sigma^g(\uparrow\uparrow - \uparrow\downarrow)}{dx_3 d\cos(\theta)} = \frac{4e_q^2 \alpha^2 \alpha_s}{Q^2} \left[(2-x_3) \log \left(\frac{1+\beta_{x_3}}{1-\beta_{x_3}} \right) - 2(2-x_3)\beta_{x_3} \right] \cos(\theta) \quad (30)$$

where $\beta_{x_3} = (1 - 4m_g^2/x_3^2 Q^2)^{1/2}$. The above result shows that there are no soft singularities. The small nonzero gluon mass is used to regulate collinear divergence. This regulator dependent term is absorbed into the polarised gluon fragmentation function at the level of full cross section, hence, the polarised gluon fragmentation function is defined in the massive gluon scheme. Since we are looking at the polarised gluon production, there is no virtual correction to this order, hence there is no UV divergences. Substituting the above equation in the parton model expression, we find the expression for $\hat{g}_1^g(x, Q^2)$ as

$$\hat{g}_1^g(x, Q^2) = 3 \frac{1}{x} \sum_q e_q^2 \int_x^1 \frac{dy}{y} \mathcal{C}_g(y, Q^2) \Delta D_g^\Lambda(x/y, Q^2) \quad (31)$$

where the coefficient function $\mathcal{C}_g(y, Q^2)$ is given by

$$\mathcal{C}_g(y, Q^2) = 2 \frac{4}{3} \frac{\alpha_s}{2\pi} \left[(2-y) \log \left(\frac{Q^2 y^2}{\Lambda^2} \right) + 2(y-2) \right] \quad (32)$$

In this paper we have discussed the importance of gluons in the polarised Λ production in e^+e^- scattering. We have analysed this using the AP equation satisfied by the quark and gluon fragmentation functions. We have

computed QCD corrections to the fragmentaion function $\hat{g}_1^\Lambda(x, Q^2)$ to order α_s in the parton model.

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Figure Captions:

1. Graph contributing to $e^-(q_1)e^+(q_2) \rightarrow \bar{q}(p_1)q(p_2)$.
2. Graphs contributing to $e^-(q_1)e^+(q_2) \rightarrow \bar{q}(p_1)q(p_2)g(p_3)$.
3. Virtual corrections to the figure.1